DIFFERENTIATION (BM)

Differentiate the following functions (1–30) with respect to x:

1. $y = 3x^5 + 3x^3 - 5$
2. $y = \dfrac{5x^4 + 6x^2 - x + 1}{x}$
3. $y = \sqrt{x} + \dfrac{1}{\sqrt{x}} + \log_e x + 2^x$
4. $y = e^x \sin x$
5. $y = x^3 \cos x$
6. $y = \dfrac{\sin x}{x}$
7. $y = \dfrac{\log_e x}{x}$
8. $y = x^4 \log_e x$
9. $y = \dfrac{e^x}{\log_e x}$
10. $y = (x^2 + 1) \sin 3x$
11. $y = \sin^2 x \cos x$
12. $y = e^x \tan x + x \log_e x$
13. $y = e^{ax} \cos (bx + c)$
14. $y = \log_e (\sin x) + \sin(\log_e x)$
15. $y = e^{\tan x} + e^{x \sin x} + e^{x^2}$
16. $y = \sin (\cos x)$
17. $y = \tan (x^2 + 3)$
18. $y = \tan x^o$
19. $y = \cos (\sin x^3)$
20. $y = \sin\left(\sqrt{1 + x^2}\right)$
21. $y = \sin^2 (3x + 4)$
22. $y = \sec^3\left(\dfrac{x}{2}\right)$
23. $y = \sin (\cos (\tan (\cot x)))$
24. $y = \cos\left(\tan\left(\sqrt{x + 1}\right)\right)$
25. $y = \cos (\sin (\log_e x))$
26. $y = \log_e x \cdot \sin (e^x)$
27. $y = \sin\left(\cos^2 (\tan^3 x)\right)$
28. $y = x^{\sin x}$
29. $y = x^{1/x}$
30. $y = x^{\log_e x}$
31. If $y = (x - a)(x - b)$, then find x for which $\dfrac{dy}{dx} = 0$
32. If $f(x) = \dfrac{x - 4}{2\sqrt{x}}$, then find $f'(4)$
33. Find $\dfrac{dy}{dx}$ where $y = \sin (x + y)$
34. Find $\dfrac{dy}{dx}$ if $x^n + y^n = a^n$
35. If $y = \sqrt{x + \sqrt{x + \sqrt{x + ...\text{to } \infty}}}$ then find the value of $(2y - 1)\dfrac{dy}{dx}$.
36. Find y' if $x^2 + y^2 + xy = 2$
37. Find y' if $y = x \cos y + y \cos x$
38. Find y' if $x = y \log_e (xy)$
39. Find y' if $xy^2 = (x + y)^4$
40. Find y' if $y = x^y$
41. If $\sqrt{\dfrac{x}{y}} + \sqrt{\dfrac{y}{x}} = 6$ then prove $\dfrac{dy}{dx} = \dfrac{x - 17y}{17x - y}$
42. Let $f(x) = (x - a)(x - b)(x - c)$ then prove that $\dfrac{f'(x)}{f(x)} = \dfrac{1}{x - a} + \dfrac{1}{x - b} + \dfrac{1}{x - c}$.
43. Find $\dfrac{dy}{dx}$ where x and y are functions of θ as $x = a \cos \theta$, $y = a \sin \theta$.
44. Find $\dfrac{dy}{dx}$ where x and y are functions of t as $x = at^2$, $y = 2at$.
45. Find $\dfrac{dy}{dx}$ where $y = a(1 - \cos \theta)$, $x = a(\theta - \sin \theta)$, (a is fixed).
46. Find $\dfrac{dy}{dx}$ where $y = e^t + \sin t$, $x = \log t$.
47. Let $y = x^5 + 2x^4 + x^3 + x^2$. Find $\dfrac{d^3y}{dx^3}$ at $x = 0$.
48. Let $y = A \sin \omega x + B \cos \omega x$, then prove that $\dfrac{d^2y}{dx^2} + \omega^2 y = 0$.
49. If $y = e^{-x} \cos x$ then show that $\dfrac{d^4y}{dx^4} + 4y = 0$.
50. Let $f(x) = (\log x)^2$ then find the value of $f''(1) + f''(e)$.

ANSWER KEY

1. $15x^4 + 9x^2$
2. $15x^2 + 6 - \frac{1}{x^2}$
3. $\frac{x + 2\sqrt{x} - 1}{2x\sqrt{x}} + 2^x \log 2$
4. $e^x (\sin x + \cos x)$
5. $3x^2 \cos x - x^3 \sin x$
6. $\frac{x\cos x - \sin x}{x^2}$
7. $\frac{1 - \log_e x}{x^2}$
8. $x^3 (1 + 4 \log_e x)$
9. $\frac{e^x (x \log_e x - 1)}{x (\log_e x)^2}$
10. $2x \sin 3x + 3(x^2 + 1) \cos 3x$
11. $2 \sin x - 3 \sin^3 x$
12. $e^x (\tan x + \sec^2 x) + \log_e x + 1$
13. $e^{ax} (a \cos (bx + c) - b \sin (bx + c))$
14. $\cot x + \frac{1}{x}\cos(\log_e x)$
15. $e^{\tan x} \sec^2 x + e^{x \sin x}(\sin x + x \cos x) + 2xe^{x^2}$
16. $-\sin x \cos (\cos x)$
17. $2x \sec^2 (x^2 + 3)$
18. $\frac{\pi}{180}\sec^2 (x^o)$
19. $-3x^2 \cos (x^3) \cdot \sin (\sin (x^3))$
20. $\frac{x}{\sqrt{1+x^2}}\cos\left(\sqrt{1+x^2}\right)$
21. $3 \sin (6x + 8)$
22. $\frac{3}{2}\sec^3\left(\frac{x}{2}\right)\tan\left(\frac{x}{2}\right)$
23. $\operatorname{cosec}^2 x \cdot \sec^2(\cot x) \cdot \sin(\tan(\cot x)) \cdot \cos(\cos(\tan(\cot x)))$
24. $-\frac{1}{2\sqrt{x+1}}\sec^2\left(\sqrt{x+1}\right)\sin\left(\tan\left(\sqrt{x+1}\right)\right)$
25. $-\frac{1}{x}\cos(\log_e x)\sin\left(\sin\left(\log_e x\right)\right)$
26. $e^x \cos\left(e^x\right)\log_e x + \frac{1}{x}\sin\left(e^x\right)$
27. $-3\tan^2 x \sec^2 x \sin\left(2\tan^3 x\right)\cos\left(\cos^2\left(\tan^3 x\right)\right)$
28. $x^{\sin x}\left(\frac{1}{x}\sin x + \cos x \log_e x\right)$
29. $x^{\frac{1}{x}}\left(\frac{1 - \log_e x}{x^2}\right)$
30. $2\frac{y}{x}\log_e x$
31. $\frac{a+b}{2}$
32. $\frac{1}{4}$
33. $\frac{(\cos(x+y))}{(1-\cos(x+y))}$
34. $-\left(\frac{x}{y}\right)^{n-1}$
35. 1
36. $\frac{-(2x+y)}{(2y+x)}$
37. $\frac{(\cos y - y \sin x)}{(1 + x \sin y - \cos x)}$
38. $\frac{(y(x-y))}{(x(x+y))}$
39. $\frac{4(x+y)^3 - y^2}{2xy - 4(x+y)^3}$
40. $\frac{yx^{y-1}}{1 - x^y \log_e x}$
43. $-\cot \theta$
44. $\frac{1}{t}$
45. $\cot\left(\frac{\theta}{2}\right)$
46. $t(e^t + \cos t)$
47. 6
50. 2